Curved pleat folding for smooth wrapping

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A technique is presented to minimize the packaged size of a flat sheet by folding it into a strip that can be wrapped around a hub. This is difficult to accomplish using regular straight creases, because of the thickness of the sheet. A curved crease pattern is used to address this problem of non-zero thickness. A mathematical formulation is developed in order to compute the required crease pattern for coiling a folded strip to any desired curvature, and specifically for coiling it into a tight spiral around a hub. This method is demonstrated with paper models. The crease computation algorithm is then extended to provide a folding strategy to efficiently package a square sheet around a central hub such that it can easily unfold or deploy. This development has applications particularly for deployable structures on small spacecraft.

***Key words*: Paper folding, curved crease, deployable structure, membrane packaging.**

1. Introduction

The problem of practically folding flat sheets into a small volume is relevant to a wide range of applications. Many everyday objects including, for example, umbrellas, maps, tents, and tarps, are designed to be opened up for use, and to be folded or rolled into a small package for storage or transport. Efficient packaging of sheets can also find application in astronautical problems, with spacecraft systems including solar panels, antennas, telescope mirrors, thermal shields, and solar sails, all of which must fit within a rocket for launch and then deploy reliably once the spacecraft is in orbit. Commercial applications include many concepts for posters, signs, and booths that can be easily transported and set up at remote locations, while military applications include communications antennas that can be easily deployed in the field. In nature, we can also see many examples of sheets that are packaged and then deployed, including flowers and leaves unfurling from their buds, and insect wings unfolding out of a cocoon or from a wing case [Vincent, 2001].

For these mechanisms described above, the goal is often to minimize the packed size of a flat sheet in such a way that it can be easily unfolded or deployed. Here we use the term *flat sheet* to refer to a membrane that may have non-zero thickness, but is inextensible with zero Gaussian curvature (also known as a *developable surface*). Possible metrics of size include the maximum

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linear dimension, and the combined linear measurement, which is the sum of the object's dimensions in three orthogonal directions. Much work has been done on similar or related problems including, for example, solar-sailing deployment experiments by Salama et al. [2003] and by Lichodziejewski et al. [2005]. In this paper, we focus on the problem of packaging sheets with non-zero thickness and leverage the mechanics of curved crease folding to provide a solution. The presented solutions will use accordion folds, which are alternating, parallel mountain and valley folds, to transform flat sheets into strips of material. For this discussion, we will refer to accordion-folded sheets as pleated sheets, where the pleat width is the distance between adjacent folds.

The work presented here is divided into two parts. In the following section, we present a general mathematical formulation for computing a curved crease pattern to coil a pleated sheet into a spiral. In section 3, we extend this formulation to the configuration of a deployable square sheet wrapped around a central hub. Section 4 concludes and proposes potential future applications that can benefit from this work.

2. Mathematical Formulation of Curved Pleat Geometry

The introduction of folds along curved creases in a flat sheet has the effect of deforming the sheet into a three-dimensional form. The geometry we are considering in this paper is a flat, inextensible sheet of non-zero, constant thickness τ, which is to be pleated using an accordion fold with spacing p. In this section, we first review previous work on folded sheets with curved creases, and we present a mathematical formulation for computing the curvature required in the creases in order to impose a folded curvature of a determined radius. Next, we discuss the simple case of no curvature, followed by the case of arbitrary radius of curvature, and finally the specific curvature required for optimal packaging around a hub.

(*a*) *Previous Work on Curved Creases*

Huffman [1976] wrote a seminal paper describing the effect of creases on paper. His work with curved creases included artistic sculptures made from paper and from polyvinyl chloride (PVC) sheets, which have been reconstructed by Demaine et al. [2010]. Similar families of three-dimensional constructs produced using curved creases were implemented not only in paper but also in thin metal and polycarbonate sheets by Koschitz et al. [2008]. By varying the shape and curvature of the creases, different forms including saddles, twists and other complex structures can be introduced into the global three-dimensional form of a folded sheet.

Kilian et al. [2008] developed a computational algorithm to determine the curved crease pattern that will allow a planar sheet to approximate a given three-dimensional surface. With a prescribed crease pattern, the shape of the folded sheet is uniquely determined. Their optimization algorithm accounts for a minimization of the bending energy in the folded sheet to ensure a physical solution. In the following development, we apply this uniqueness property of curved creases to the problem of packaging a deployable membrane. Specifically, the crease curvature is used to enable tight wrapping of a pleated sheet.

(*b*) *Pleat With No Curvature*

The simplest case is of a pleat that is to be kept straight, or in other words, at an infinite radius of curvature. In this case, the accordion creases are straight lines and are parallel, each spaced at a distance p from adjacent creases. The result is a folded strip of thickness nt, where n is the number of folded segments.

If one were to bend this folded strip, the layers would try to shear: the outer layers would be in tension while the inner layers would be in compression. With the outer layers unable to stretch, the inner layers would buckle and wrinkle.

(*c*) *General Pleat Curvature*

In order to curve a pleated strip at a specified radius r, we can determine an explicit geometric relationship between r and the radius of curvature of the crease, R. In the general case, we choose a reference line, which is the crease that follows the specified curvature. On each side of this reference line, the sheet spans a width $w_+(\xi)$ and $w_-(\xi)$, respectively, where ξ is the coordinate specifying path length along the crease. The geometry of the flat sheet is shown in figure 1a and the corresponding geometry of the folded and curved pleat is shown in figure 1b. The thickness of the folded pleat above and below the reference line is denoted h_+ and h_-, respectively, corresponding to the unfolded width w_+ and w_-. For a small crease length $d\xi$ along the reference line, the crease length at the outer edge of the sheet is

$$d\xi_+ = \frac{R_+}{R} d\xi = \left(\frac{R + w_+}{R}\right) d\xi. \tag{2.1}$$

This must remain equal in the folded and flat geometries, such that

$$\left(\frac{R + w_+}{R}\right) d\xi = \left(\frac{r + h_+}{r}\right) d\xi. \tag{2.2}$$

We can then solve for the crease radius of curvature,

$$R = \left(\frac{w_+}{h_+}\right) r. \tag{2.3}$$

Now, a substitution can be made to solve for the ratio of width to pleat thickness. With the assumption of constant membrane thickness τ and pleat width p, we have

$$h_+ = \left(\frac{w_+}{p}\right) \tau. \tag{2.4}$$

This can be rearranged to be

$$\frac{w_+}{h_+} = \frac{p}{\tau}, \tag{2.5}$$

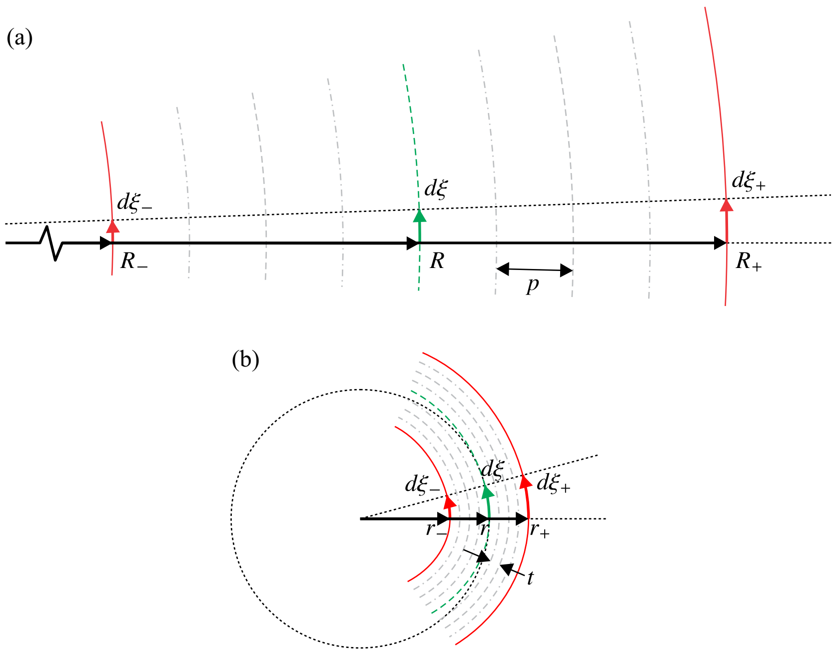
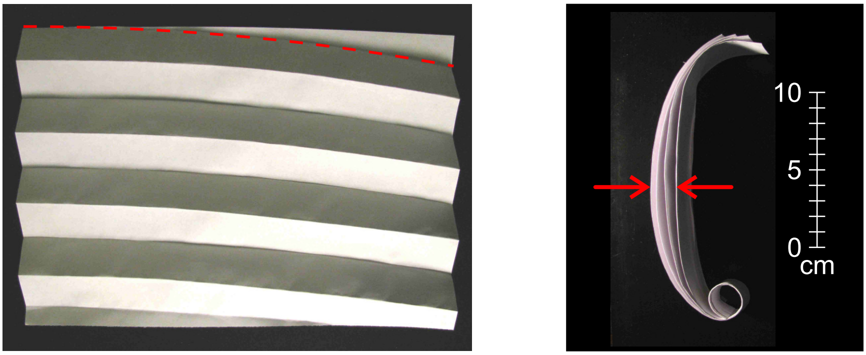


Figure 1. Curved crease geometry showing the relationship between the (a) flat and (b) folded configurations of the sheet. Valley folds are denoted with dashed lines and mountain folds with dash-dotted lines; the pattern can also be reversed. In this example with a sheet of thickness t, there are four pleats of width p on each side of the central reference line. A small length along the reference line is shown in both the flat and folded configurations as $d\xi$. The radius of curvature at this point of the folded reference line is r, corresponding to a crease curvature of radius R. The equivalent distances at the edge of the sheet are shown as $d\xi_-$ and $d\xi_+$, with folded radii r_- and r_+ and crease radii R_- and R_+, respectively. (Online version in colour.)



(a) (b)

Figure 2. Paper folded with constant curvature creases. (a) Unfolded configuration showing the crease pattern. The reference line is indicated with the dashed line at the top of the sheet, and has a constant radius of curvature of 1.83 m. The other creases are spaced 25.4 mm apart and are alternately mountain and valley folds. (b) Folded configuration with variable radius of curvature. At the bottom of the photo, the pleat is curved at a radius of approximately 8 mm, allowing the pleat to lie flat. At the centre of the photo, the pleat is curved at a larger radius of curvature, and does not lie flat as indicated by the arrows. Both photos are shown at the same scale. (Online version in colour.)

and finally,

$$R = \left(\frac{w_+}{h_+}\right) r = \left(\frac{p}{\tau}\right) r, \tag{2.6}$$

which provides a constant scaling from coil radius r to crease curvature radius R for a constant thickness membrane. Likewise, the geometry on the other side of the reference line yields the same result by equating $d\xi_-$ in the flat and folded configurations, providing a consistent radius of curvature for the reference line. The remaining crease lines are defined as contours around the reference line spaced by the pleat width p, which remains well behaved assuming $p/\tau \gg 1$. Note that for infinitely thin sheets, the ratio p/τ approaches infinity, resulting in straight creases as expected.

An implementation of a crease pattern with constant curvature is shown in figure 2. Here, a letter-sized (216 mm × 279 mm) sheet of regular copy paper (with weight 75 g/m^2) was folded with creases spaced 25.4 mm apart. The top-most crease was used as the reference line with a radius of curvature of 1.83 m. The thickness of the paper is 0.11 mm, which results in a nominal pleat curvature of approximately 8 mm. The photo on the right shows the curved pleat positioned with a variable curvature, and it is clear that the pleat does not lie flat for curvatures with radius larger than this nominal value.

(*d*) *Spiral Wrapping of a Single Pleat Around a Hub*

In order to wrap a pleat around a hub, we require the radius of curvature to gradually increase as the pleat is wrapped tightly over the previous layer. To compute the desired coil radius of the pleat, the problem is separated into two parts. The first part, where the pleat is making its first revolution around the hub, is computed as an Archimedes' spiral such that after one revolution, the inner layer of the pleat lies flat on the outer layer at the initial radius of the hub. The remaining length of the pleat is constrained to maintain continuous contact between the inner layer of the pleat and the outer layer of the previous revolution. This can be solved in general for a given pleat defined by thickness $h_-(\xi)$ and $h_+(\xi)$, and for a given initial radius $r(0)$.

Depending on the geometry of the sheet, different mathematical descriptions of the sheet width and corresponding pleat thickness can be used. For a rectangular sheet with the reference line centred on one edge, we have $w_+(\xi) \approx w_-(\xi) = \text{constant}$, and with the reference line starting at a corner, we have $w_+(\xi) \approx w_-(\xi) \approx \xi$. The algorithm is not very sensitive to this parameter locally, since the error in pleat thickness is typically a small fraction of the pleat radius of curvature. However, the integrated error in pleat thickness can result in a desired pleat curvature that does not spiral at the correct rate, resulting in suboptimal packing. If the designed crease is too straight, the pleat will not coil as tightly, or will buckle and crumple as with the straight crease pattern. If the designed crease is too curved, the pleat will not lie flat, as was demonstrated in figure 2.

For the first revolution, the Archimedes' spiral can be written as

$$r(\theta) = r(0) + \frac{r(2\pi) - r(0)}{2\pi}\theta = r(0) + b\theta, \tag{2.7}$$

where b is the spiral rate parameter. Here, we have replaced the path length coordinate ξ with an angular coordinate θ defined relative to the hub. The closed-form expression for the path length ζ along an Archimedes' spiral of the form $r = b\phi$ is given by Weisstein [2010] as

$$\zeta(\phi) = \frac{1}{2}b\left[\phi\sqrt{1+\phi^2} + \ln\left(\phi + \sqrt{1+\phi^2}\right)\right]. \tag{2.8}$$

For a spiral with initial radius $r(0)$, we can use the substitution $\phi = \theta + r(0)/b$ to solve for the path length $\xi(\theta) = \zeta(\phi) - \zeta(r(0)/b)$. This path length along the spiral can be expressed after some simplification in terms of the angular coordinate as

$$\begin{aligned}\xi(\theta) = &\frac{(b\theta + r(0))\sqrt{b^2 + (b\theta + r(0))^2}}{2b} \\ &+ \frac{1}{2}b\ln\frac{\sqrt{b^2 + (b\theta + r(0))^2} + b\theta + r(0)}{\sqrt{b^2 + r(0)^2} + r(0)} \\ &- \frac{r(0)\sqrt{b^2 + r(0)^2}}{2b}.\end{aligned} \tag{2.9}$$

We can solve numerically for b such that

$$r(2\pi) = r(0) + h_+(0) + h_-(\xi(2\pi)). \tag{2.10}$$

For the remainder of the pleat, we propagate r to maintain contact with the previous layer by integrating the equation

$$\dot{\xi} = \sqrt{\dot{r}^2 + r^2}, \tag{2.11}$$

such that

$$r(\theta) = r(\theta - 2\pi) + h_+(\xi(\theta - 2\pi)) + h_-(\xi(\theta)). \tag{2.12}$$

The integration can be done numerically for given sheet and coil parameters. Figure 3 shows the spiral-wrapped pleat geometry and associated crease reference line for a 203 mm × 267 mm sheet of 75 g/m^2 paper, with pleats spaced 25.4 mm apart and an initial coil radius of 5 mm. The computed crease radius of curvature ranges from 1.27 m to 2.54 m along the length of the crease, as plotted in figure 3c. Figure 4 contains photos of the crease pattern in its unfolded and folded configurations. A comparison of figure 3a to figure 4b shows that the dimensions of the physical implementation of the coil are well predicted by the computation. The inner and outer coiled radii of the folded paper match the theoretical prediction to within 5% and is limited by the flexibility inherent in the material.

3. Extension to Square Sheets with Central Hubs

The work presented in the previous section can be applied to the problem of packaging a membrane around a central hub by solving for the required pleat crease curvature for multiple pleated regions. In this section, we start by reviewing previously developed crease patterns for packaging and deploying thin sheets. We then present a modified crease pattern that extends the computation demonstrated above, by partitioning the sheet into regions that can be treated as individual pleats. Finally, we show a physical implementation of this new folding strategy using paper.

(*a*) *Previous Work on Deployable Sheets*

Kobayashi et al. [1998] proposed a folding pattern modelled after hornbeam and beech leaves. This pattern is composed of two pleated halves joined at a central spine at a constant angle. The pattern is equivalent to a single row of fundamental units from Miura's "developable double corrugation surface" [1985], also known as the Miura fold. The use of non-orthogonal folds in this strategy enables two key features. First, it spatially staggers the crease nodes where several fold lines connect and where the folded membrane is therefore thickest. This allows the pattern to be more robust to increases in scale, by reducing the strain introduced at the crease nodes. Second, it couples the two-dimensional folding (and unfolding) process into a single continuous phase

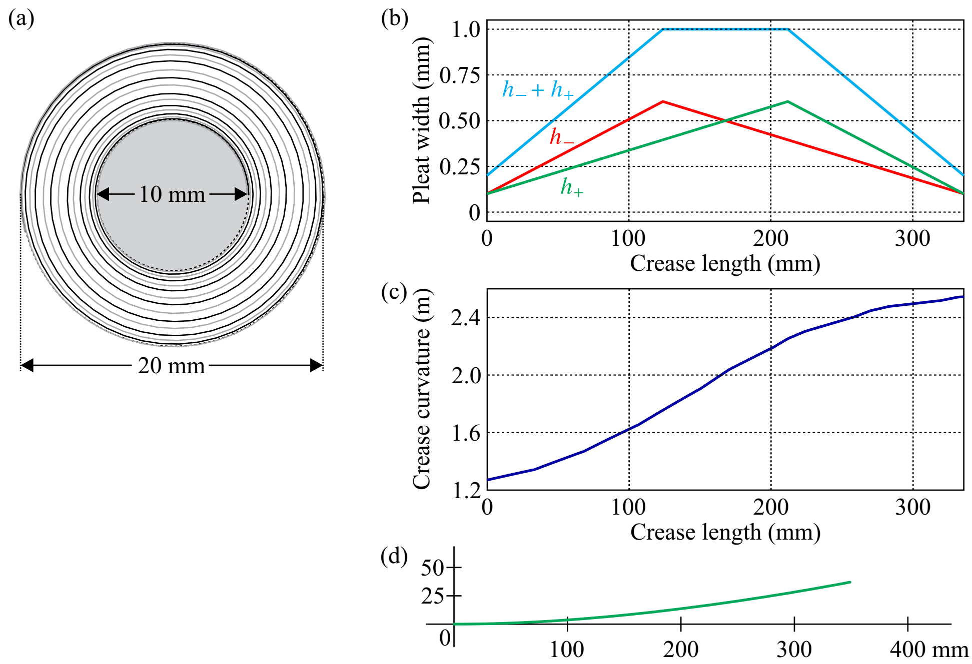
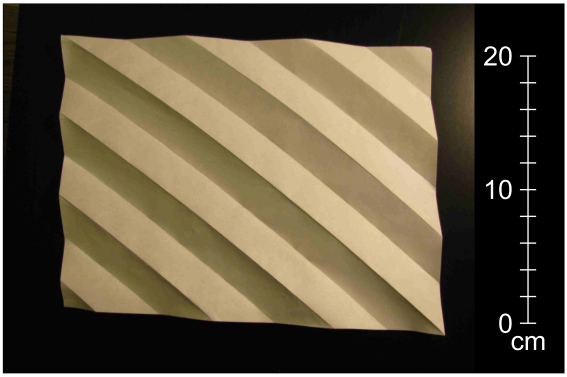
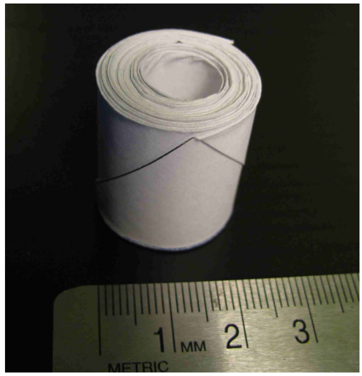


Figure 3. Theoretical crease design for a single pleat wrapped tightly around a hub. (a) Folded pleat curvature computed for a rectangular sheet of paper pleated along a diagonal, with an initial coil radius of 5 mm. The reference line is shown in black, with the edge of the pleat shown in grey. Dotted lines are reference lines at constant radius. (b) Pleat width functions showing the thickness of the folded pleat on either side of the reference line and in total. The functions are piecewise linear and derived from empirical measurements. (c) Crease curvature as a function of the length along the crease. (d) Geometry of the reference line plotted with the left end aligned with the horizontal axis. (Online version in colour.)



(a)



(b)

Figure 4. Paper folded with crease pattern designed for spiral wrapping as shown in figure 3. (a) Unfolded configuration showing the crease pattern. The reference line is the crease extending from the top-left to the bottom-right of the sheet using the geometry shown in figure 3c. The other creases are spaced 25.4 mm apart and are alternately mountain and valley folds. (b) Folded configuration with the pleat tightly wrapped starting at the nominal radius of curvature. The final radius of the coiled pleat is approximately 2 mm, as predicted by the reference design shown in figure 3a. The two photos are not shown at the same scale. (Online version in colour.)

rather than two discrete steps. This allows the sheet to be deployed more smoothly and with fewer separate actuators.

This beech-leaf folding technique was adapted by De Focatiis and Guest [2002] and by Furuya et al. [2005] by tiling the crease pattern in the four quadrants of a square membrane and six equiangular sectors of a circular membrane, respectively. De Focatiis proposed two configurations (*leaf-out* and *leaf-in*), which allow either radial spokes or the edge of the membrane to be free of creases. Furuya's technique is similar to the leaf-out pattern, but with each panel shifted (akin to the panels of a camera shutter) to allow placement of a hub in a hole in the centre of the sheet.

An alternative strategy for wrapping a circular or polygonal sheet around a central hub was studied by Guest and Pellegrino [1992]. This work focuses on wrapping the sheet around a polygonal hub, using discrete folds to transition from face to face. This strategy differs from the work of De Focatiis and of Furuya in that the height of the coil increases as the sheet is wrapped. Additionally, it proposes adaptations in the fold pattern to account for non-zero thickness of the sheet being wrapped. By incorporating curved creases into a fold pattern, we present an approach that allows the assumption of infinitely thin sheets in the De Focatiis and Furuya folding techniques to be relaxed.

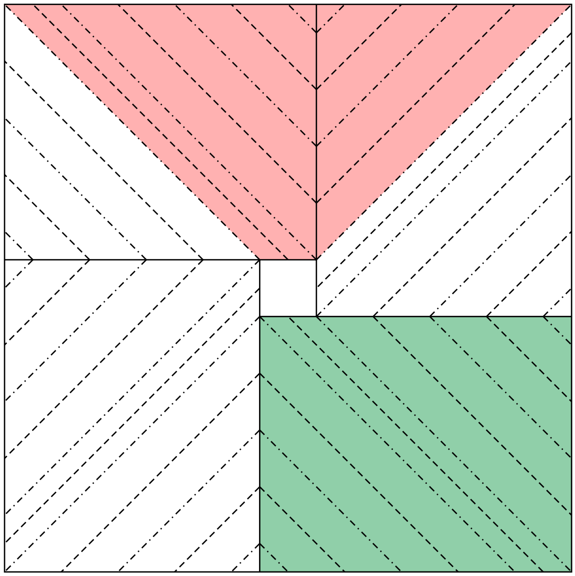


Figure 5. Folding pattern for a square sheet using straight creases. Valley folds are denoted with dashed lines and mountain folds with dash-dotted lines; the pattern can also be reversed. Note that creases lie along the diagonals of the square. The shaded trapezoidal region at the top of the design is equivalent to Kobayashi's beech-leaf pattern which was used by De Focatiis, and to the *L*-fold panel in Furuya's design. The shaded rectangular region in the lower right is equivalent to the *Z*-fold panel in Furuya's design, and is composed only of parallel pleats. (Online version in colour.)

(*b*) *Spiral Wrapping of a Square Sheet Around a Central Hub*

The crease patterns used by De Focatiis and Guest [2002] and Furuya et al. [2005] were adapted to accommodate a square membrane with a central hub, as shown in figure 5. In particular, the leaf-out pattern was selected because it includes straight radial segments where structural supports can be placed most easily. The square section was rotated by 45 degrees from the De Focatiis design, to place these radial segments along the diagonals of the square. Additionally, the offset position of each quadrant was adopted from the Furuya design to accommodate the placement of a hub in the centre of the sheet. Finally, in order to accommodate an arbitrarily sized square hole for the hub, the width of the central pleats around the reference line are sized to match the hole.

This crease pattern can be interpreted in two ways. The first is that it is composed of four quadrants (the upper red shaded region in the figure) that are each folded using the beech leaf *V*-shaped pattern. The second is that it is composed of four quadrants (the lower green shaded region in the figure) that are simply pleated in an accordion fold along the diagonal. This second interpretation facilitates the use of our development from the previous section to wrap this more complex pleat geometry around the central core. To account for the membrane thickness, the crease pattern is modified such that each pleat is designed to curve at a radius such that the layers will lie flat.

The algorithm presented in section 2(d) is for a single pleat being coiled around a hub. For a square sheet, there will be four pleats comprising the full membrane. The equations above can be modified for k-fold rotational symmetry, where for this square design, k is 4. For the first $1/k$ revolutions, equation 2.10 is rewritten as

$$r(2\pi/k) = r(0) + h_+(0) + h_-(\xi(2\pi/k)), \tag{3.1}$$

and for the remainder of the pleat, equation 2.12 is rewritten as

$$r(\theta) = r(\theta - 2\pi/k) + h_+(\xi(\theta - 2\pi/k)) + h_-(\xi(\theta)). \tag{3.2}$$

(*c*) *Implementation of Spiral-Wrapped Square Sheet*

Including these modifications to the crease computation, a numerical solver was implemented in MATLAB using a 1st-order Euler method with a fixed step size of 3.6 degrees. Crease patterns were computed for 0.14 mm thick parcel paper. The central pleats are 15.9 mm wide to form a central square hole with a width of 45 mm, and the remaining pleats are 70 mm wide. These dimensions were selected to provide a proof of concept of a deployable sheet that would be compliant with the 10 cm × 10 cm × 10 cm size constraint of the CubeSat standard, which is commonly used for small spacecraft.

Parcel paper was used to test the crease algorithm because of its additional thickness compared to regular copy paper. Figure 6a shows the coiled pleat geometry computed using MATLAB for a 1 m × 1 m sheet with a thickness of 0.14 mm using a hub radius of 20 mm, which provides a reasonable minimum coil curvature from which the membrane must deploy. The pleat thickness functions $h_+(\xi)$ and $h_-(\xi)$ were determined empirically by folding a quadrant using straight creases and measuring the thickness of the pleat with calipers. The crease pattern derived from this coil geometry is shown in figure 6b for $p/\tau = 300$, using the reference crease shown in figure 6d. The computed crease radius of curvature ranges from 6 m to 12 m along the length of the crease, as plotted in figure 6c. The coiled radius of the square membrane matched the theoretical coil geometry to within 10%. The creases are not noticeably curved at the scale shown, but the deviation from the straight crease pattern enabled the sheet to be successfully wrapped around a hub, as shown in figure 7 while an equivalent pattern with straight creases was unable to wrap neatly.

4. Conclusions

We have presented the development of a new technique to compute crease curvature in order to obtain desired three-dimensional behaviour in a pleated strip. This technique was applied to provide a specific packaging strategy for square sheets that was shown to work in practice. The technique can be extended to sheets of different sizes, thicknesses, and materials. In future work, a useful study would be to characterize local folding behaviour and to explore the limits

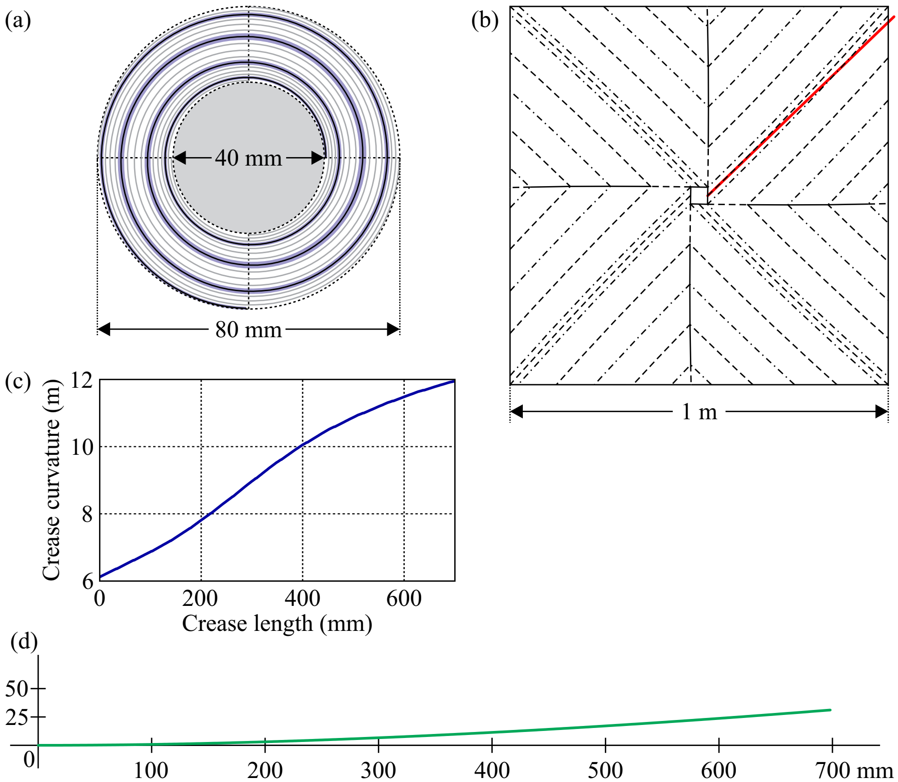


Figure 6. Theoretical crease design for a square sheet wrapped tightly around a central hub. (a) Folded pleat curvature computed for a 1 m × 1 m square sheet of paper, with an initial coil radius of 20 mm. The reference line for one quadrant is shown in black, with the thickness of the pleat shown as the shaded region around the black spiral. The equivalent reference lines for the other three quadrants are shown in grey. Dotted lines are reference lines at constant radius. (b) The complete crease pattern as shown in figure 5 modified with curved creases. Valley folds are denoted with dashed lines and mountain folds with dash-dotted lines. A straight reference line is drawn in the top right. (c) Crease curvature as a function of the length along the crease. (d) Geometry of the reference line for a single quadrant plotted with the left end aligned with the horizontal axis. (Online version in colour.)


(a) (b)

Figure 7. Parcel paper folded with crease pattern designed for spiral wrapping as shown in figure 6. (a) Unfolded configuration showing the crease pattern. The four quadrants are constructed separately and attached only at the hub and the edge of the sheet. The diagonal of each quadrant is folded with the reference line using the geometry shown in figure 6c. (b) Folded configuration with the pleat tightly wrapped starting at the nominal radius of curvature around a solid hub. The two photos are not shown at the same scale. (Online version in colour.)

of this technique for thicker sheets or narrower pleats. Additionally, adapting this technique to accommodate curved sheets (for example, paraboloids for reflectors) and other materials (including non-uniform sheets) would enhance the utility of the work.

The packaging technique described here can provide benefits in volume-constrained applications. One particular area where optimal packaging is critical is in small spacecraft deployable systems. There is currently much interest in deployable high-power solar arrays that use thin film photovoltaic materials on flexible membranes for spacecraft in the 1–10 kg mass range, and the increasing number of objects in space has recently highlighted the problem of space debris. A deployable solar array, or drag sail that could reduce the lifetime of orbiting spacecraft, would be an ideal application for the work in this paper.

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